

EFFECT OF THERMODIFFUSION ON INTENSITY OF SURFACE
BURN-OUT IN A HETEROGENEOUS TURBULENT BOUNDARY LAYER

É. P. Volchkov, P. V. Nikitin,
and E. I. Sinaiko

UDC 536.244:533.15

On the basis of an analysis of the binary diffusion equation in the viscous sublayer of the turbulent boundary layer an estimate is given of the effect of thermodiffusion on the concentration of components at an impervious wall, on the permeability of the wall, and on the intensity of heating of the reactive surface. Experimental results are presented on the burn-out of a graphite wall in mixtures of air with nitrogen, argon, and helium.

In studies of a boundary layer involving chemical reactions because of the complex nature of the effect a number of assumptions are made concerning the chemical reaction rates and the composition and properties of the gas mixture. In this case, as a rule, only concentration diffusion is considered, where the diffusion flux of the material is determined by the concentration gradient.

In the context of boundary layer theory barodiffusion can be neglected, since the pressure is constant across the boundary layer. Moreover, in a binary boundary layer under certain conditions the diffusion thermoeffect and thermodiffusion [1-3] can become important.

The diffusion thermoeffect is characterized by heat transfer produced by a concentration gradient. The presence of this effect leads to the fact that the temperature of the adiabatic surface can differ significantly from the temperature outside the boundary layer.

Thermodiffusion represents diffusion mass transfer arising in the presence of a temperature gradient within the gas mixture. The molecules of the heavier gas usually diffuse in the direction of the heat flux, while the lighter molecules diffuse in the opposite direction. Here the thermodiffusion intensity depends both on the characteristics of the temperature field and on the ratio of the molecular weights of the components under examination. Thermodiffusion leads to some separation of the components of the gas mixture.

Thus, in a nonisothermal binary boundary layer thermodiffusion can lead to a considerable change in concentrations at the wall.

The diffusion equation for a binary mixture, described through weight concentrations, with thermodiffusion taken into account has the form [1, 2]

$$j_1 = \rho V_1 K_1 = -\rho D_{12} \left[\frac{\partial K_1}{\partial y} + k_T \frac{M_1 M_2}{M^2} \frac{\partial \ln T}{\partial T} \right] \quad (1)$$

Here j_1 , K_1 , and V_1 are the mass flux, weight concentration of the first component, and velocity; T is the temperature; ρ the gas density; M_1 , M_2 , and M are the molecular weights of the individual components and of the gas mixture; D_{12} and D_T are the concentration and thermal diffusion coefficients; and $k_T = D_T/D_{12}$ is the thermodiffusion ratio.

The thermodiffusion ratio depends strongly on the relative content of the two components and on the ratios of masses and diameters of the molecules. It is therefore sometimes convenient to use the thermodiffusion constant α , which depends rather weakly on the concentration of the components:

$$\alpha = \frac{k_T}{n_1 n_2}, \quad \alpha (1 - K_1) K_1 = k_T \frac{M_1 M_2}{M^2} \quad (2)$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 119-125, September-October, 1972. Original article submitted March 30, 1972.

© 1974 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for \$15.00.

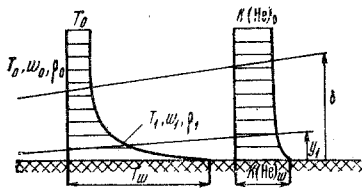


Fig. 1

Since the diffusion equations for a multiple-component mixture are very complex we shall first examine a heterogeneous binary boundary layer in the absence of chemical reactions. We shall determine the concentrations of the components at the wall taking thermodiffusion into account. Then using the results obtained we will determine the permeability parameter b_1^* at the reactive wall which characterizes the intensity of chemical erosion of the surface.

A diagram of the problem under consideration is presented in Fig. 1. A wall with a surface temperature T_w is blown upon by a heterogeneous gas mixture (henceforth we will examine a mixture of air with helium or argon) having a temperature T_0 in the undisturbed current.

A turbulent boundary layer with a thickness δ develops at the wall. In this case the laminar sublayer has a thickness y_1 with gas parameters ρ_1 , w_1 , and T_1 at the sublayer boundary. As the analysis conducted in [4] shows, for Prandtl and Schmidt numbers $P < 1$ and $S < 1$ the difference in velocity at the boundaries of the dynamic, thermal, and diffusion laminar sublayers need not be taken into account.

If the temperature were constant over the thickness of the boundary layer, the concentrations of the components of the gas mixture would be constant. The temperature does vary smoothly over the entire thickness of the layer from T_0 at the outer boundary to T_w at the surface, which leads to the development of thermodiffusion.

The characteristics of turbulent transfer prevail in the region $y_1 < y < \delta$. Here the turbulent diffusion coefficients D^* are many times larger than the coefficients of molecular concentration D_{12} and thermal diffusion D_T . In such a case, despite the presence of a temperature gradient, thermodiffusion in the turbulent center can be neglected. The concentrations of the individual components of the mixture in the region of the turbulent center can be taken as constant to a first approximation; i.e., the concentrations at the boundary of the laminar sublayer are equal to their values in the undisturbed stream:

$$K(i)_{y=y_1} = K(i)_0$$

The processes of molecular transport are established in the region of the laminar sublayer (for $y < y_1$). The maximum temperature gradients occur here, which leads to the development of thermodiffusion and a gradient in the concentrations of the individual components of the mixture.

In the film theory the flow in the laminar sublayer is replaced by the Couette model of flow on the assumption that the variation in parameters in the flow direction is negligibly small compared with their variation in the transverse direction. In this case it follows from the continuity equation that the transverse flow of material is constant across the sublayer cross section:

$$j = \rho w_y = \text{const} = j_w$$

and if an impermeable wall is considered, $j = 0$ in the laminar sublayer.

From Eq. (1), written with (2) in consideration, at an impermeable wall:

$$\frac{\partial K}{\partial y} = -\alpha K(1-K) \frac{\partial \ln T}{\partial T} \quad (3)$$

it follows that in this case because of thermodiffusion in the laminar sublayer a concentration gradient arises such that the concentration diffusion is balanced by the thermodiffusion.

For a small change in concentration due to thermodiffusion one can assume that $\alpha \approx \text{const}$ and, integrating Eq. (3) over the thickness of the laminar sublayer, obtain

$$\int_{K_w}^{K_1} \frac{dK}{K(1-K)} = - \int_{T_w}^{T_1} \frac{\alpha}{T} dT \quad (4)$$

Hence, with the condition that the concentration of a component at the sublayer boundary is equal to its value in the undisturbed current $K_1 = K_0$, we get the equation

$$\frac{K_0(1-K_w)}{K_w(1-K_0)} = \left(\frac{T_w}{T_1} \right)^\alpha \quad (5)$$

or an expression for the concentration of the component at the wall:

$$K_w = \frac{K_0}{K_0 + (1-K_0)(T_w/T_1)^\alpha} \quad (6)$$

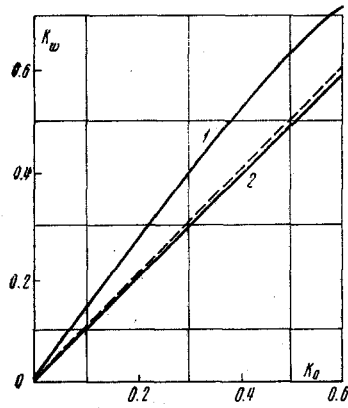


Fig. 2

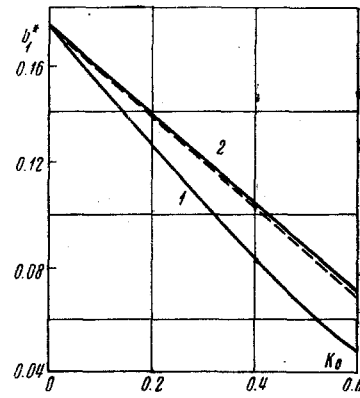


Fig. 3

To determine the concentration of the components at the wall it is necessary to know the temperature T_1 at the boundary of the viscous sublayer. As shown in [4], for the flow of a compressible gas at a thermally isolated surface and for heat emission from the wall to the gas, the value of the universal coordinate η_1 is close to its value in an incompressible fluid; i.e., at the boundary of the laminar sublayer one can take

$$\varphi_1 = \eta_1 = 11.6 \quad \text{OR} \quad \varphi_1 = w / v^* = 11.6 \quad (7)$$

Let us express the dynamic velocity through the coefficient of friction at the wall:

$$v^* = \sqrt{\frac{\tau_w}{\rho}} = w_0 \sqrt{\frac{\rho_0}{\rho}} \sqrt{\frac{c_f}{2}} \quad (8)$$

It follows from Eqs. (7) and (8) with the condition of similarity of the velocity and temperature profiles that

$$\omega = \frac{T_w - T_1}{T_w - T_0} = 11.6 \sqrt{\frac{\rho_0}{\rho}} \sqrt{\frac{c_f}{2}} \quad (9)$$

The ratio of densities is determined from the equation of state and the condition of similarity of the temperature and concentration profiles. From the latter relation one can determine the temperature T_1 at the boundary of the laminar sublayer.

Let us examine the conditions under which the experiments on a graphite surface which is being burned out [5, 6] were conducted:

$$T_0 \approx 290^\circ \text{K}, \quad T_w \approx 2000^\circ \text{K} \\ 10^5 < R_x < 3 \cdot 10^6, \quad c_f/2 = \text{St} P^{0.6} \approx 1.2 \cdot 10^{-3}$$

$c_f/2 = \tau_w / \rho_0 w_0^2$ is the coefficient of friction, $\text{St} = j_c / \rho_0 w_0 b_1$ is the Stanton diffusion number, $R_x = \rho_0 w_0 x / \mu_0$ is the Reynolds number along the length of the surface, and $S = \mu / \rho D_{12}$ is the Schmidt number.

Solving Eq. (9) relative to the temperature at the boundary of the laminar sublayer we find that under these conditions

$$T_1 = 840^\circ \text{K}, \quad \omega_1 \approx 0.68$$

Now the concentration of the components of the gas mixture at the wall in the absence of chemical reactions and a transverse flow of material can be determined from Eq. (6). Calculations were conducted for mixtures of air with helium and air with argon. Values of the thermodiffusion constant α as a function of the concentration of the components were chosen in accordance with the data of [2, 3].

The results of calculations of helium and argon concentrations at the wall as a function of their concentration in the main current and taking thermodiffusion into account are presented in Fig. 2. As seen, the helium concentration (curve 1) can be increased by 20–35% at an impermeable wall for the conditions under examination. The argon concentration (curve 2) decreases, but not significantly (by 2.5%). In the absence of the thermodiffusion effect the concentrations of the components at the wall and in the main current are the same (dashed straight line).

In an analysis of the effect of thermodiffusion and diffusion thermoeffect on heat exchange at a porous wall in [3, 7] it is recommended that these effects be taken into account in determining the equilibrium tem-

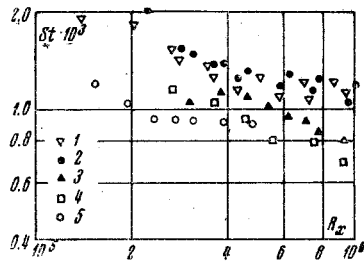


Fig. 4

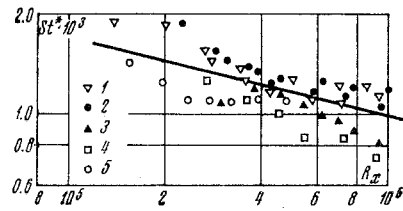


Fig. 5

perature of the wall. Here the heat emission coefficient is conservative with respect to the thermodiffusion effect. An analogous model was used by the authors in [5] to analyze the efficiency of a gas screen at a surface which is being burned out. It was shown that the laws of thermal mass exchange remain conservative if the calculations are made with respect to the "equilibrium" concentration of the components at the wall (with respect to the oxidant concentration determined on the assumption of a gas screen with the absence of chemical reactions at the surface). In this case the intensity of burn-out of a graphite wall was determined from the following equation:

$$j_c = \rho_0 w_0 St^* b_1^* \quad (10)$$

Here St^* is the Stanton diffusion number. In the case where the reaction $C + O \rightarrow CO$ occurs at the surface the permeability parameter is determined by the equation

$$b_1^* = \frac{3}{4} K(O)_w^* \quad (11)$$

where $K(O)_w^*$ is the oxygen concentration at the wall under the conditions examined in the absence of chemical reactions.

Thus, if the laws of thermal mass exchange remain conservative with respect to thermodiffusion, the effect of thermodiffusion on the burn-out intensity will only show up through the permeability parameter b_1^* .

The results of calculations of the permeability parameter from Eqs. (6) and (11) for the burn-out of graphite in mixtures of air with helium (curve 1) or with argon (curve 2) are presented in Fig. 3 as a function of the argon or helium concentration in the current. The dashed line represents a calculation without taking thermodiffusion into account. As seen, in a mixture of air with helium the thermodiffusion effect is considerably stronger than in a mixture of air with argon. Here in the first case thermodiffusion leads to a reduction in the burn-out intensity [by 30% for $K(He)_0 \approx 0.6$], while in the second case it leads to an insignificant increase [by 2.5% for $K(Ar)_0 \approx 0.6$].

In the experimental arrangement which was described in [5] test data were obtained on the burn-out of PG-50 graphite in a current of various gas mixtures. The burn-out intensity was studied in a current of air diluted by nitrogen (uniform boundary layer) and in mixtures of air with helium and with argon. The mass flow of the gas mixture in the tests was $\rho_0 w_0 \approx 100 \text{ kg/m}^2 \cdot \text{sec}$, the temperature of the gas current was $T_0 \approx 290^\circ\text{K}$, and the temperature of the graphite wall was $T_w = 1880\text{--}2025^\circ\text{K}$.

The results of these experiments, analyzed without considering the thermodiffusion effect, are presented in Fig. 4. In this case the experimental value of the Stanton number was determined from (10) while the permeability parameter was determined from the oxygen concentration in the main current:

$$St = j_c / \rho_0 w_0 b_1, \quad b_1 = \frac{3}{4} K(O)_0 \quad (12)$$

As seen, the test results in mixtures of air with nitrogen $K(N)_0 = 0.77\text{--}0.874$ (points 1) and in mixtures of air with argon $K(Ar)_0 = 0.31\text{--}0.56$ (points 2) practically coincide apart from the dependence on the concentration of components in the main current. The test data obtained on the mixtures of air with helium are stratified as a function of the helium concentration in the main current and are distributed lower (by 40%). The designations of the points in Fig. 4 correspond to the following helium concentrations in the main current: points 3: 6.2%, 4: 13%, 5: 35%.

The same test data analyzed with the effect of thermodiffusion taken into account are presented in Fig. 5. In this case the experimental value of the Stanton number was determined from the equation

$$St^* = j_c / \rho_0 w_0 b_1^*$$

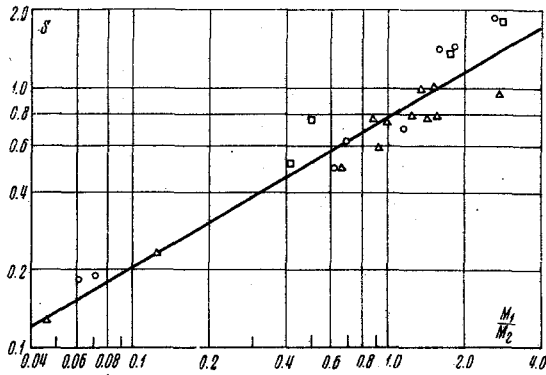


Fig. 6

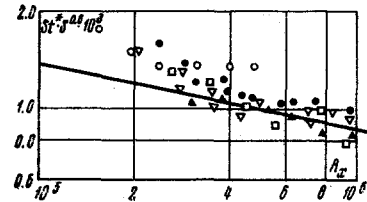


Fig. 7

where b_1^* was found from Eq. (11) with allowance for the effect of thermodiffusion on the oxygen concentration $K(O)_w^*$ at the wall in the absence of chemical reactions (Fig. 3). The test data are compared with the dependence [5]

$$St = 0.029R_x^{-0.2}S^{-0.6}\Psi^{0.8}(\mu_w/\mu_0)^{0.2} \quad (13)$$

with the condition that $P \approx S = 0.71$. Here $\Psi = (St/St_0)_{R^*}$ is the relative law of thermal mass exchange, taking into account the nonisothermal effect and transverse flow of material at the wall, and μ_w and μ_0 are the gas viscosity at the wall and in the main current.

It must be kept in mind that in the case of such a heterogeneous mixture in addition to thermodiffusion a change in the Schmidt number S as a function of the concentration of the components can be important. The dependence of the Schmidt number on the ratio of the molecular weights of the diffusing gas M_1 to the molecular weight of the medium M_2 is presented in Fig. 6. The values of S were determined from the coefficients of binary diffusion presented in [8-10]. As seen, the available data correlate satisfactorily in this simple form, and one can use an exponential dependence (for $0.05 < M_1/M_2 < 5$) in a first approximation to determine the S number:

$$S \approx 0.76 (M_1 / M_2)^{0.57} \quad (14)$$

In determining the permeability parameter b_1^* at a graphite surface which is being burned out, diffusion equations for oxygen and carbon [5] are examined in which the coefficients of diffusion of oxygen and carbon monoxide in the gas mixture are involved. In determining the S number the multicomponent mixture will be considered as a certain binary mixture, where oxygen O_2 (or CO) diffuses in a mixture having a molecular weight M_2 .

The results of experiments on the intensity of graphite burn-out in mixtures of air with nitrogen, argon, and helium, constructed with thermodiffusion and the S number taken into account, are presented in Fig. 7. The designations of the test points are the same as in Fig. 4.

In this analysis the experimental data obtained in a heterogeneous boundary layer (mixture of air with argon and helium) correlate satisfactorily with the results of experiments in a uniform boundary layer (mixture of air with nitrogen) and are described by Eq. (13).

For the experimental conditions used the maximum effect of thermodiffusion on the burn-out intensity did not exceed 20%. However, for other ratios of components and under more nonisothermal conditions the thermodiffusion effect can be more important.

LITERATURE CITED

1. K. E. Grew and T. L. Ibbs, *Thermal Diffusion in Gases*, Cambridge University Press (1952).
2. Baron, "Thermodynamic generalized forces in boundary layers," *Raketa. Tekhn.*, No. 7 (1962).
3. Sparrow, Minkowycz, and Eckert, "Diffusion thermoeffects in an air current in the vicinity of the critical point during blowing at a boundary layer of gases with different molecular weights," *Raketa. Tekhn. i Kosmonavtika*, No. 4 (1964).
4. Yu. V. Lapin, *The Turbulent Boundary Layer in Supersonic Gas Streams* [in Russian], Nauka, Moscow (1970).

5. É. P. Volchkov, E. G. Zaulichnyi, A. I. Leont'ev, and K. I. Sinaiko, "Interaction of a graphite surface with a turbulent gas current under essentially nonisothermal conditions and in the presence of a screen of neutral gas," *Teplofiz. Vys. Temp.*, 8, No. 1 (1970).
6. A. I. Leont'ev, É. P. Volchkov, E. G. Zaulichnyi, and E. I. Sinaiko, "Experimental determination of rate of carrying off a graphite under essentially nonisothermal conditions," *Fizika Goreniya i Vzryva*, 3, No. 2 (1967).
7. S. S. Kutateladze, *Fundamentals of Heat Exchange Theory* [in Russian], Nauka, Novosibirsk (1970).
8. J. Hirschfelder, C. Curtiss, and R. Bird, *Molecular Theory of Gases and Liquids* [Russian translation], Izd. Inostr. Lit., Moscow (1961).
9. E. R. Eckert and R. M. Drake, *Heat and Mass Transfer*, McGraw-Hill (1959).
10. Brettshneider, in: *Properties of Gases and Liquids* [Russian translation], Khimiya, Moscow-Leningrad (1966).